

# Galvano-rotational effect induced by electroweak interactions in pulsars

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**Abstract.** We study electroweakly interacting particles in rotating matter. The existence of the electric current along the axis of the matter rotation is predicted in this system. This new galvano-rotational effect is caused by the parity violating interaction between massless charged particles in the rotating matter. We start with the exact solution of the Dirac equation for a fermion involved in the electroweak interaction in the rotating frame. This equation includes the noninertial effects. Then, using the obtained solution, we derive the induced electric current which turns out to flow along the rotation axis. We study the possibility of the appearance of the galvano-rotational effect in dense matter of compact astrophysical objects. The particular example of neutron and hypothetical quark stars is discussed. It is shown that, using this effect, one can expect the generation of toroidal magnetic fields comparable with poloidal ones in old millisecond pulsars. We also briefly discuss the generation of the magnetic helicity in these stars. Finally we analyze the possibility to apply the galvano-rotational effect for the description of the asymmetric neutrino emission from a neutron star to explain pulsars kicks.

**Keywords:** neutron stars, magnetic fields, gravity

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## 1 Introduction

The importance of noninertial effects for various areas in modern physics cannot be underestimated. Some of the examples of these effects are mentioned in ref. [1]. One of the most common manifestations of noninertial effects is the description of physical processes in a rotating frame. One can expect the appearance of additional interesting phenomena if, besides the matter rotation, there is a parity violating interaction in the system. In the present work we will show that, in this situation, an electric current flowing along the rotation axis can be induced. We shall call this phenomenon as the new *galvano-rotational effect* (GRE).

Previously the generation of an electric current due to nontrivial topological effects was studied mainly in connection to the chiral magnetic effect (CME) [2]. In that case a nonzero current can be induced in the system of massless particles embedded in an external magnetic field [3], provided there is an imbalance between left and right particles. Recently, in ref. [4], we showed that the electric current can be generated even at zero chiral imbalance if charged particles are involved in the parity violating interaction.

We will apply the new GRE in astrophysical media to generate a toroidal magnetic field (TMF) inside a compact star. Although stellar TMFs cannot be observed directly, they are an internal ingredient of various astrophysical objects. For example, the most plausible explanation of the 22 yr solar cycle is the oscillation between poloidal and toroidal components of the solar magnetic field [5]. Moreover, purely poloidal or toroidal stellar magnetic fields were shown in refs. [6, 7] to be unstable. Thus TMF is inherent to a magnetized star. In this work we show how TMF can be generated in an old millisecond pulsar basing on GRE.

Since macroscopic fluxes of electroweakly interacting particles are produced by GRE, we can try to apply this effect to explain linear velocities of millisecond pulsars. It is known from astronomical observations [8] that pulsars possess great linear velocities. Nevertheless physical processes underlying pulsar kicks are still unclear. It might be reasonable to use GRE to account for pulsars kicks due to, e.g., anisotropic neutrino emission since linear velocities of pulsars were reported in ref. [9] to be correlated with their angular velocities.

This paper is organized in the following way. First, in section 2, we briefly describe the Standard Model interaction between leptons and quarks in flat space-time. Then, in section 3, we derive the Dirac equation for a fermion which interacts electroweakly with a rotating background matter. For this purpose we write down this Dirac equation in the corotating frame, using the method of an effective curved space-time. The exact solution of the Dirac equation for an ultrarelativistic fermion, accounting for the noninertial effects, is obtained in section 3. In section 4, we establish GRE, which, in this situation, consists in the appearance of the electric current along the rotation axis. We calculate this current in section 4 using the exact solution of the Dirac equation obtained in section 3. In section 5, we apply GRE for the generation of TMF and the magnetic helicity in compact rapidly rotating stars. Finally, in section 6, we try to use GRE to produce anisotropic neutrino emission from pulsars to explain their great linear velocities.

In section 7, we summarize our results and compare them with the findings of other authors. Some details of the derivation of the electric current in rotating matter are provided in appendix A.

## 2 Electroweak interaction of fermions in flat space-time

In this section we shall briefly remind the description of the electroweak interaction between leptons and quarks in flat space-time.

Let us consider a medium consisting of electrons, electron neutrinos  $\nu_e$  as well as  $u$  and  $d$  quarks. We shall assume that quarks are both in confined states, forming nucleons, and hypothetical free particles. The effective Lagrangian for the electroweak interaction in this system in the Fermi approximation has form [10],

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left[ J^\mu J_\mu^\dagger + K^\mu K_\mu \right], \quad (2.1)$$

where  $G_F \approx 1.17 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi constant,  $J^\mu$  is the charged current, and  $K^\mu$  is the neutral current.

In the considered system of elementary particles, the charged current has the form,

$$J_\mu = V_{ud} \bar{\psi}_u \gamma_\mu^L \psi_d + \bar{\psi}_{\nu_e} \gamma_\mu^L \psi_e, \quad (2.2)$$

where  $\psi_{u,d}$  are the wave functions of  $u$  and  $d$  quarks,  $\psi_{\nu_e,e}$  are the wave functions of  $\nu_e$  and an electron,  $\gamma_\mu^L = \gamma_\mu (1 - \gamma^5)/2$ ,  $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$  are the Dirac matrices,  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ , and  $V_{ud} \approx 0.97$  is the element of the Cabbibo-Kobayashi-Maskawa matrix. The neutral current can be expressed as

$$K_\mu = \sum_{f=u,d,e,\nu_e} \left[ \epsilon_f^L \bar{\psi}_f \gamma_\mu^L \psi_f + \epsilon_f^R \bar{\psi}_f \gamma_\mu^R \psi_f \right], \quad (2.3)$$

where  $\gamma_\mu^R = \gamma_\mu (1 + \gamma^5)/2$  and

$$\begin{aligned} \epsilon_e^L &= -\frac{1}{2} + \xi, & \epsilon_{\nu_e}^L &= \frac{1}{2}, & \epsilon_u^L &= \frac{1}{2} - \frac{2}{3}\xi, & \epsilon_d^L &= -\frac{1}{2} + \frac{1}{3}\xi, \\ \epsilon_e^R &= \xi, & \epsilon_{\nu_e}^R &= 0, & \epsilon_u^R &= -\frac{2}{3}\xi, & \epsilon_d^R &= \frac{1}{3}\xi. \end{aligned} \quad (2.4)$$

Here  $\xi = \sin^2 \theta_W \approx 0.23$  and  $\theta_W$  is the Weinberg angle.

Test particle	Background particles	$V_L$	$V_R$
electron	nucleons	$-\frac{G_F}{\sqrt{2}} [n_n - n_p(1 - 4\xi)] (2\xi - 1)$	$-\frac{G_F}{\sqrt{2}} [n_n - n_p(1 - 4\xi)] 2\xi$
$\nu_e$	nucleons & electrons	$-\frac{G_F}{\sqrt{2}} [n_n - 2n_e]$	0
$u$ quark	$d$ quarks	$-\frac{G_F}{\sqrt{2}} n_d (1 - \frac{8}{3}\xi + \frac{16}{9}\xi^2 - 2 V_{ud} ^2)$	$\frac{G_F}{\sqrt{2}} n_d (\frac{4}{3}\xi - \frac{16}{9}\xi^2)$
$d$ quark	$u$ quarks	$-\frac{G_F}{\sqrt{2}} n_u (1 - \frac{10}{3}\xi + \frac{16}{9}\xi^2 - 2 V_{ud} ^2)$	$\frac{G_F}{\sqrt{2}} n_u (\frac{2}{3}\xi - \frac{16}{9}\xi^2)$

**Table 1.** The values of the effective potentials  $V_{L,R}$  in eq. (2.5) for various channels of the scattering of a test fermion off background particles. Here  $n_e$  is the electron density,  $n_{n,p}$  are the densities of neutrons and protons, and  $n_{u,d}$  are the densities of  $u$  and  $d$  quarks.

In this section we shall consider the case when background fermions are at rest and unpolarized. While averaging the currents in eqs. (2.2) and (2.3) over the Fermi-Dirac distributions  $\langle \dots \rangle$ , in this case we get that only  $\langle \psi_f^\dagger \psi_f \rangle = n_f \neq 0$ , where  $n_f$  is the invariant number density of these fermions. The quantities  $\langle \bar{\psi}_f \gamma \psi_f \rangle = 0$  and  $\langle \bar{\psi}_f \gamma^\mu \gamma^5 \psi_f \rangle = 0$  are vanishing since they are proportional to the macroscopic velocity and the polarization.

After averaging over the ensemble of background particles, we can rewrite eq. (2.1) in the form,

$$\mathcal{L}_{\text{eff}} = -\bar{\psi} [\gamma_0^L V_L + \gamma_0^R V_R] \psi, \quad (2.5)$$

where  $\psi$  is the wave function of a test fermion which undergoes a scattering off background particles and the effective potentials  $V_{L,R}$  are given in table 1 for any scattering channels.

When we consider the electron and  $\nu_e$  scattering off nucleons,  $u$  and  $d$  quarks are confined inside neutrons or protons. In the case of electron-nucleons interaction, only the neutral current contributes to eq. (2.5). If we study the scattering  $u$  quarks off  $d$  quarks, and vice versa, as well as the  $\nu_e$  scattering off background fermions, both the charged and the neutral currents give the contributions to eq. (2.5). To derive the expression for  $V_L$  for  $(ud)$ ,  $(du)$ , and  $(\nu_e e)$  interactions on the basis of eq. (2.2), we use the Fierz transformation. We also note that we consider the  $\nu_e$  interaction with electroneutral matter where  $n_e = n_p$ .

### 3 Dirac equation for a fermion interacting with rotating matter

In this section we shall find the exact solution of the Dirac equation for a fermion interacting with a rotating matter by means of the electroweak forces. The obtained solution includes noninertial effects.

Let us discuss the interaction of a fermion with matter rotating with the constant angular velocity  $\omega$ . In this case we cannot directly apply the results of section 2 taking  $\langle \bar{\psi}_f \gamma \psi_f \rangle \sim \mathbf{v}_f \neq 0$ , where  $\mathbf{v}_f = (\boldsymbol{\omega} \times \mathbf{r})$  is the fermions velocity, while averaging over the ensemble of background particles. In the situation, when matter moves with an acceleration, one should account for possible noninertial effects.

Nevertheless we can still choose a noninertial reference frame where matter is at rest. Assuming that matter is unpolarized, we get that only  $\langle \psi_f^\dagger \psi_f \rangle \neq 0$  in this reference frame. Thus, formally we can use the effective potentials derived in section 2. For the first time this approach was put forward in ref. [11], where the neutrino interaction with a rotating matter was studied.

It is known that the description of a particle in an accelerated frame is analogous to the motion of this particle in the curved space-time or the interaction with an effective gravitational field. For example, when we study the motion in the rotating frame, the interval takes the form [12],

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 - \omega^2 r^2) dt^2 - dr^2 - 2\omega r^2 dt d\phi - r^2 d\phi^2 - dz^2, \quad (3.1)$$

where  $g_{\mu\nu}$  is the metric tensor of the effective gravitational field. Here we use the cylindrical coordinates  $x^\mu = (t, r, \phi, z)$ .

Using eq. (2.5), we get that the Dirac equation for a test fermion, with the mass  $m$ , involved in the parity violating interaction and moving in a curved space-time, has the form (see also ref. [11]),

$$[i\gamma^\mu(x)\nabla_\mu - m]\psi = \gamma_\mu(x) \left\{ \frac{V_L^\mu}{2} [1 - \gamma^5(x)] + \frac{V_R^\mu}{2} [1 + \gamma^5(x)] \right\} \psi, \quad (3.2)$$

where  $\gamma^\mu(x)$  are the coordinate dependent Dirac matrices,  $\nabla_\mu = \partial_\mu + \Gamma_\mu$  is the covariant derivative,  $\Gamma_\mu$  is the spin connection,  $\gamma^5(x) = -(i/4!) E^{\mu\nu\alpha\beta} \gamma_\mu(x) \gamma_\nu(x) \gamma_\alpha(x) \gamma_\beta(x)$ ,  $E^{\mu\nu\alpha\beta} = \varepsilon^{\mu\nu\alpha\beta} / \sqrt{-g}$  is the covariant antisymmetric tensor in curved space-time,  $g = \det(g_{\mu\nu})$ , and  $V_{L,R}^\mu$  are the effective potentials. Note that, since we choose a corotating frame, then  $V_{L,R}^0 \equiv V_{L,R} \neq 0$  and  $V_{L,R}^i = 0$ , where  $V_{L,R}$  are given in table 1. Analogous Dirac equation was also discussed in ref. [13].

One can check that, using the following vierbein vectors:

$$e_0^\mu = (1, 0, -\omega, 0), \quad e_1^\mu = (0, 1, 0, 0), \quad e_2^\mu = (0, 0, 1/r, 0), \quad e_3^\mu = (0, 0, 0, 1), \quad (3.3)$$

the metric tensor in eq. (3.1) can be diagonalized,  $\eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu}$ , where  $\eta_{ab} = \text{diag}(1, -1, -1, -1)$  is the metric in a locally Minkowskian frame.

Let us introduce the constant Dirac matrices in a locally Minkowskian frame by  $\gamma^{\bar{a}} = e_a^\mu \gamma^\mu(x)$ . As shown in ref. [11],  $\gamma^5(x) = i\gamma^{\bar{0}}\gamma^{\bar{1}}\gamma^{\bar{2}}\gamma^{\bar{3}} = \gamma^{\bar{5}}$  does not depend on coordinates. The spin connection in the Dirac eq. (3.2) has the form [14],

$$\Gamma_\mu = -\frac{i}{4} \sigma^{ab} \omega_{ab\mu}, \quad \omega_{ab\mu} = e_a^\nu e_{b\nu;\mu}, \quad (3.4)$$

where  $\sigma_{ab} = (i/2)[\gamma_{\bar{a}}, \gamma_{\bar{b}}]_-$  are the generators of the Lorentz transformations in a locally Minkowskian frame and the semicolon stays for the covariant derivative. The explicit calculation on the basis of eq. (3.4) shows that the nonzero components of the connection one-form  $\omega_{ab} = \omega_{ab\mu} dx^\mu$  are

$$\omega_{12\mu} = -\omega_{21\mu} = (\omega, 0, 1, 0). \quad (3.5)$$

Using eqs. (3.4) and (3.5) we get that  $i\gamma^\mu(x)\Gamma_\mu = i\gamma^{\bar{1}}/2r$ .

Using the definition of  $\gamma^{\bar{a}}$ , the Dirac eq. (3.2) takes the form,

$$\begin{aligned} \mathcal{D}\psi &= \left( \gamma^{\bar{0}} - \omega r \gamma^{\bar{2}} \right) \left( V_V - V_A \gamma^{\bar{5}} \right) \psi, \\ \mathcal{D} &= \left[ i\gamma^{\bar{0}} (\partial_0 - \omega \partial_\phi) + i\gamma^{\bar{1}} \left( \partial_r + \frac{1}{2r} \right) + i\gamma^{\bar{2}} \frac{\partial_\phi}{r} + i\gamma^{\bar{3}} \partial_z - m \right], \end{aligned} \quad (3.6)$$

where  $V_{V,A} = (V_L \pm V_R)/2$  are the vector and axial parts of the effective potentials. Note that the operator  $\mathcal{D}$  in eq. (3.6) is analogous to that recently studied in ref. [15].

Since eq. (3.6) does not explicitly depend on  $t$ ,  $\phi$ , and  $z$ , we shall look for its solution in the form,

$$\psi = \exp(-iEt + iJ_z\phi + ip_z z) \psi_r, \quad (3.7)$$

where  $\psi_r = \psi_r(r)$  is the wave function depending on the radial coordinate. The values of  $J_z$  in eq. (3.7) were found in ref. [16] to be  $\pm 1/2, \pm 3/2, \dots$

It is convenient to rewrite eq. (3.6) as

$$[\gamma^{\bar{a}} Q_a - m + V] \psi_r = 0, \quad (3.8)$$

where  $Q^a = q^a - q_{\text{eff}} A_{\text{eff}}^a$ ,  $q_{\text{eff}}$  is the effective electric charge,  $q^a = (E + J_z\omega - V_V, -i\partial_r, 0, p_z)$ ,

$$A_{\text{eff}}^a = \left(0, \frac{i}{2q_{\text{eff}}r}, \frac{1}{q_{\text{eff}}} \left[V_V\omega r - \frac{J_z}{r}\right], 0\right) \quad (3.9)$$

is the potential of the effective electromagnetic field, and  $V = V_A (\gamma^{\bar{0}} - \omega r \gamma^{\bar{2}}) \gamma^{\bar{5}}$ .

Let us look for the solution of eq. (3.8) in the form,  $\psi_r = [\gamma^{\bar{a}} Q_a + m - V] \Phi$ , where  $\Phi$  is the new spinor. The equation for  $\Phi$  reads

$$\begin{aligned} & \left[ \left( \partial_r + \frac{1}{2r} \right)^2 + (E + J_z\omega - V_V)^2 - \left( \frac{J_z}{r} - V_V\omega r \right)^2 + \left( V_V\omega + \frac{J_z}{r^2} \right) \Sigma_3 \right. \\ & \quad \left. + 2V_A \gamma^{\bar{5}} \left[ \left( \frac{J_z}{r} - V_V\omega r \right) \omega r - (E + J_z\omega - V_V) + \frac{\omega}{2} \Sigma_z \right] - p_z^2 \right. \\ & \quad \left. + V_A^2 (1 - \omega^2 r^2) + 2mV_A (\gamma^{\bar{0}} - \omega r \gamma^{\bar{2}}) \gamma^{\bar{5}} - m^2 \right] \Phi = 0, \quad (3.10) \end{aligned}$$

where  $\Sigma_3 = \gamma^{\bar{0}} \gamma^{\bar{3}} \gamma^{\bar{5}}$ .

The solution of eq. (3.10) can be found for ultrarelativistic particles. In the limit  $m \rightarrow 0$ , we can represent  $\Phi = v\varphi$  in eq. (3.10), where  $\varphi = \varphi(r)$  is a scalar function and  $v$  is a constant spinor satisfying  $\Sigma_3 v = \sigma v$  and  $\gamma^{\bar{5}} v = \chi v$ , with  $\sigma = \pm 1$  and  $\chi = \pm 1$ , since both  $\Sigma_3$  and  $\gamma^{\bar{5}}$  now commute with the operator of eq. (3.10).

Let us first study left particles,  $(1 + \gamma^{\bar{5}})\psi = 0$ , corresponding to  $\chi = +1$ . The case of right particles with  $\chi = -1$  can be studied analogously. Using the new variable  $\rho = |V_L|\omega r^2$  we can write the equation for  $\varphi_\sigma$  as

$$\left[ \rho \partial_\rho^2 + \partial_\rho - \frac{1}{4\rho} \left( l + \frac{\sigma}{2} \text{sgn}(V_L) - \frac{1}{2} \right)^2 - \frac{\rho}{4} - \frac{1}{2} \left( l - \frac{\sigma}{2} \text{sgn}(V_L) - \frac{1}{2} \right) + \kappa \right] \varphi_\sigma = 0, \quad (3.11)$$

where

$$\begin{aligned} \kappa = & \frac{1}{4|V_L|\omega} \left[ E^2 + 2E(J_z\omega - V_L) - p_z^2 - m^2 + (V_L)^2 + J_z^2\omega^2 + V_L\omega\sigma \right] \\ & + \frac{1}{2} \left( l - \frac{\sigma}{2} \text{sgn}(V_L) - \frac{1}{2} \right). \end{aligned} \quad (3.12)$$

In eq. (3.11) we choose  $J_z = (1/2 - l)\text{sgn}(V_L)$ , where  $l = 0, \pm 1, \pm 2, \dots$

Assuming that  $\varphi_\sigma \rightarrow 0$  at  $r \rightarrow \infty$ , we get that  $\varphi_+ = I_{N,s}$  and  $\varphi_- = I_{N-1,s}$ , for  $V_L > 0$ , as well as  $\varphi_+ = I_{N-1,s}$  and  $\varphi_- = I_{N,s}$ , for  $V_L < 0$ . Here  $N = 0, 1, 2, \dots$ ,  $s = N - l$ , and

$I_{N,s} = I_{N,s}(\rho)$  is the Laguerre function<sup>1</sup>. The energy spectrum can be found if we take that  $\kappa = N$  in eq. (3.12). We can present  $E$  is the form,

$$E = V_L + \left(l - \frac{1}{2}\right) \omega \text{sgn}(V_L) \pm \mathcal{E}, \quad \mathcal{E} = \sqrt{p_z^2 + m^2 + 4|V_L|N\omega}. \quad (3.13)$$

One can see that the neutrino energy depends on the sign of  $V_L$ . Note that one should understand  $m \neq 0$  in eq. (3.13) in the perturbative sense.

The total radial wave function  $\psi_r$  can be found in the explicit form if we choose the spinors  $v_\sigma$  as

$$v_+^T = (1, 0, 0, 0), \quad v_-^T = (0, 1, 0, 0). \quad (3.14)$$

In eq. (3.14) we assume that the Dirac matrices are in the chiral representation [17],

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^{\bar{k}} = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (3.15)$$

where  $\sigma_k$  are the Pauli matrices.

Using eq. (3.14), we get the radial wave functions corresponding to different spin projections,

$$\psi_r^\pm = \begin{pmatrix} 0 \\ \eta_\pm \end{pmatrix}, \quad \eta_+ = \Pi \begin{pmatrix} \varphi_+ \\ 0 \end{pmatrix}, \quad \eta_- = \Pi \begin{pmatrix} 0 \\ \varphi_- \end{pmatrix}, \quad (3.16)$$

where

$$\Pi = \begin{pmatrix} p_z - E - J_z \omega + V_L & -iR_\pm \\ -iR_\mp & -p_z - E - J_z \omega + V_L \end{pmatrix}, \quad (3.17)$$

and

$$\begin{aligned} R_+ &= \partial_r + \frac{1}{2r} + \frac{J_z}{r} - V_L \omega r = \sqrt{|V_L| \omega \rho} \left( 2\partial_\rho - \frac{l-1}{\rho} - 1 \right), \\ R_- &= \partial_r + \frac{1}{2r} - \frac{J_z}{r} + V_L \omega r = \sqrt{|V_L| \omega \rho} \left( 2\partial_\rho + \frac{l}{\rho} + 1 \right). \end{aligned} \quad (3.18)$$

In eq. (3.17) the upper signs stay for  $V_L > 0$  and the lower ones for  $V_L < 0$ .

The properly normalized two component spinors in eq. (3.16) have the form,

$$\eta_+ = C_+ \begin{pmatrix} \mp [\mathcal{E} \mp p_z] I_{N,s} \\ -2i\sqrt{|V_L| \omega N} I_{N-1,s} \end{pmatrix}, \quad \eta_- = C_- \begin{pmatrix} 2i\sqrt{|V_L| \omega N} I_{N,s} \\ \mp [\mathcal{E} \pm p_z] I_{N-1,s} \end{pmatrix}, \quad (3.19)$$

for  $V_L > 0$ , and

$$\eta_+ = C_+ \begin{pmatrix} \mp [\mathcal{E} \mp p_z] I_{N-1,s} \\ 2i\sqrt{|V_L| \omega N} I_{N,s} \end{pmatrix}, \quad \eta_- = C_- \begin{pmatrix} -2i\sqrt{|V_L| \omega N} I_{N-1,s} \\ \mp [\mathcal{E} \pm p_z] I_{N,s} \end{pmatrix}, \quad (3.20)$$

for  $V_L < 0$ . The signs in eqs. (3.19) and (3.20) are correlated with the sign in eq. (3.13).

It should be noted that the spinors  $\eta_+$  and  $\eta_-$  in eqs. (3.19) and (3.20) are linearly dependent as it should be for ultrarelativistic particles. Thus, one can use any independent pair of  $\eta_+$  and  $\eta_-$ . As usual, we shall attribute  $\eta_-$  with the upper sign to a particle degree of freedom and  $\eta_+$  with the lower sign to antiparticles. This choice of independent spinors

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<sup>1</sup>The definition of the Laguerre function is given, e.g., in ref. [11].

Values of $V_{L,R}$	Allowed value of $p_z$
Left particles	
$V_L > 0$	$p_z < 0$
$V_L < 0$	$p_z > 0$
Left antiparticles	
$V_L > 0$	$p_z > 0$
$V_L < 0$	$p_z < 0$
Right particles	
$V_R > 0$	$p_z > 0$
$V_R < 0$	$p_z < 0$
Right antiparticles	
$V_R > 0$	$p_z < 0$
$V_R < 0$	$p_z > 0$

**Table 2.** Allowed values of  $p_z$  in the ground state  $N = 0$  for different values of the effective potentials  $V_{L,R}$  for left and right particles and antiparticles.

is convenient for  $N > 0$ . If  $N = 0$ , one can better use  $\eta_+$  for  $V_L > 0$  and  $\eta_-$  for  $V_L < 0$  as independent degrees of freedom.

Using the normalization condition for the total wave function,

$$\int \psi_{N,s,p_z}^\dagger(x) \psi_{N',s',p'_z}(x) \sqrt{-g} d^3x = \delta_{NN'} \delta_{ss'} \delta(p_z - p'_z), \quad (3.21)$$

which includes the dependence on  $p_z$  and  $\phi$ , we get the coefficients  $C_\sigma$  in eqs. (3.19) and (3.20) as

$$C_\sigma^2 = \frac{|V_L| \omega}{2\pi \mathcal{E}(\mathcal{E} \mp \sigma p_z)}, \quad (3.22)$$

which is valid for any sign of  $V_L$ .

On the basis of eqs. (3.19) and (3.20) one can notice that, at  $N = 0$ ,  $p_z$  is correlated with the particle helicity. Using eq. (3.13) with  $m = 0$  as well as eqs. (3.19) and (3.20), one can find the possible values of  $p_z$  at  $N = 0$ . For the convenience, they are listed in table 2. Note that at  $N > 0$ ,  $-\infty < p_z < +\infty$ .

It should be noted that  $p_z$  in table 2 is a formal quantum number. The physical value of  $p_z$  for antiparticles is opposite to that shown in table 2:  $p_z^{(\text{phys.})} = -p_z$ . Otherwise the electric charge would not be conserved.

Right particles can be treated in the same way as left ones. That is why we just present only the final results. The expression for the energy has the same structure as eq. (3.13) with the replacement  $V_L \rightarrow V_R$ . In eq. (3.16) one has  $\psi_r^\pm = (\xi_\pm, 0)^T$ , where

$$\xi_+ = C_+ \begin{pmatrix} \mp [\mathcal{E} \pm p_z] I_{N,s} \\ 2i\sqrt{|V_R|\omega N} I_{N-1,s} \end{pmatrix}, \quad \xi_- = C_- \begin{pmatrix} -2i\sqrt{|V_R|\omega N} I_{N,s} \\ \mp [\mathcal{E} \mp p_z] I_{N-1,s} \end{pmatrix}, \quad (3.23)$$

for  $V_R > 0$ , and

$$\xi_+ = C_+ \begin{pmatrix} \mp [\mathcal{E} \pm p_z] I_{N-1,s} \\ -2i\sqrt{|V_R|\omega N} I_{N,s} \end{pmatrix}, \quad \xi_- = C_- \begin{pmatrix} 2i\sqrt{|V_R|\omega N} I_{N-1,s} \\ \mp [\mathcal{E} \mp p_z] I_{N,s} \end{pmatrix}, \quad (3.24)$$



for  $V_R < 0$ . The argument of the Laguerre functions is  $\rho = |V_R|\omega r^2$  now. The new normalization constant in eq. (3.23) and (3.24) reads

$$C_\sigma^2 = \frac{|V_R|\omega}{2\pi\mathcal{E}(\mathcal{E} \pm \sigma p_z)}. \quad (3.25)$$

The signs in eqs. (3.23)-(3.25) are correlated with the signs in the expression for the energy.

As in the case of left particles, for right fermions we have that  $-\infty < p_z < +\infty$  at  $N > 0$ . The allowed values of  $p_z$  at  $N = 0$  are shown in table 2.

#### 4 Induced electric current along the rotation axis

In this section we show that there is a nonzero induced electric current flowing along the rotation axis in the system of electroweakly interacting particles. In our calculation we shall use the exact solution of the Dirac equation obtained in section 3.

In section 3 we already mentioned that there is a correlation between  $p_z$  and the helicity at  $N = 0$ . Thus one expects that there can be macroscopic fluxes of particles in the rotating matter. Let us first examine this issue for left fermions. We shall calculate the mean hydrodynamic currents of particles and antiparticles with respect to the coordinates  $x^\mu$  in the rotating frame. These currents have the form,

$$j_{L,f,\bar{f}}^\mu = \sum_{N,s=0}^{\infty} \int dp_z \bar{\psi} \gamma^\mu(x) \psi \rho_{f,\bar{f}}^L(\mathcal{E} \pm V_L), \quad (4.1)$$

where  $\rho_{f,\bar{f}}^L(E) = \{\exp(\beta[E \mp \mu_L]) + 1\}^{-1}$  is the Fermi-Dirac distribution for fermions, with the lower sign staying for antifermions,  $\beta = 1/T$  is the reciprocal temperature of the fermion gas, and  $\mu_L$  is the chemical potential of left particles. The spinor in eq. (4.1) corresponds to the exact solution of the Dirac equation in eqs. (3.19) and (3.20). Note that, for the first time, this method for the calculation of the current was proposed in ref. [3].

We will be interested in the expression for  $j_{L,f}^\mu$  linear in  $\omega$ . That is why we use  $\mathcal{E} + V_L$  instead of the total particle energy, cf. eq. (3.13), in the distribution function. The contribution of the noninertial part of the energy  $\sim \omega \text{sgn}(V_{L,R})(l - 1/2)$ , see eq. (3.13), to the currents is computed in appendix A. We should study only  $j_{L,f}^3$  since it is this component of the current that is linear in  $\omega$ .

Using the orthogonality of Laguerre functions,

$$\sum_{s=0}^{\infty} I_{N,s}(\rho) I_{N',s}(\rho) = \delta_{NN'}, \quad (4.2)$$

as well as eqs. (3.3), (3.7), (3.15), (3.16), (3.19), (3.20), and (3.22) we get that

$$\sum_{s=0}^{\infty} \bar{\psi} \gamma^3(x) \psi = \pm \frac{p_z}{\mathcal{E}} \frac{|V_L|\omega}{\pi}, \quad (4.3)$$

where the upper sign stays for particles and the lower one for antiparticles.

On the basis of eq. (4.3) and table 2 we find that only the lowest level  $N = 0$  contributes to the current. Finally we get that

$$j_{L,f}^3 = -\frac{V_L\omega}{\pi} \int_0^\infty dp \rho_f^L(p + V_L). \quad (4.4)$$

Analogously to eq. (4.4) we can obtain the following expression:

$$j_{L\bar{f}}^3 = -\frac{V_L\omega}{\pi} \int_0^\infty dp \rho_{\bar{f}}^L(p - V_L), \quad (4.5)$$

which is valid for antifermions.

The contribution to the hydrodynamic current from right fermions is analogous to eqs. (4.4) and (4.5). It is

$$j_{Rf}^3 = \frac{V_R\omega}{\pi} \int_0^\infty dp \rho_f^R(p + V_R), \quad (4.6)$$

for particles, and

$$j_{R\bar{f}}^3 = \frac{V_R\omega}{\pi} \int_0^\infty dp \rho_{\bar{f}}^R(p - V_R), \quad (4.7)$$

for antiparticles. Here  $\rho_{f,\bar{f}}^R(E)$  are the distributions of right particles and antiparticles which can be obtained from  $\rho_{f,\bar{f}}^L(E)$  by replacing  $\mu_L \rightarrow \mu_R$ , where  $\mu_R$  is the chemical potential of right fermions.

Now, using eqs. (4.4)-(4.7), we can obtain the expression for the third component of the electric current as  $J^3 = q_f (j_{Lf}^3 - j_{L\bar{f}}^3 + j_{Rf}^3 - j_{R\bar{f}}^3)$ , where  $q_f$  is the electric charge of the fermion  $f$  including the sign. For example,  $q_e = -e$  for an electron,  $q_u = 2e/3$  for an  $u$  quark, and  $q_d = -e/3$  for a  $d$  quark. Here  $e > 0$  is the absolute value of the elementary electric charge. In the expression for  $J^3$ , we use the convention that the direction of the electric current coincides with the motion of the positive electric charge. Finally we get for the electric current,

$$\mathbf{J} = \frac{q_f\omega}{\pi} (V_R\mu_R - V_L\mu_L), \quad (4.8)$$

where we restore vector notations. We remind that in eq. (4.8) we keep only the terms linear in  $\omega$  and  $V_{L,R}$ . Some details of the derivation of eq. (4.8) from eqs. (4.4)-(4.7) are provided in appendix A.

We can attribute the existence of the induced electric current in rotating matter, where the parity violating interaction is present, to the new GRE; cf. section 1.

## 5 Generation of TMF and magnetic helicity in a pulsar

In this section we apply GRE for the calculation of TMF inside a pulsar. We also briefly consider the generation of the magnetic helicity in a compact rotating star.

If we consider a rapidly rotating compact astrophysical object, like a neutron star (NS) or even a hypothetical quark star (QS), then the mechanism described in section 4 will induce the electric current along the rotation axis of such a star. We shall suppose that this current forms a closed circuit connected somewhere at the stellar surface. Thus, using the Maxwell equation  $(\nabla \times \mathbf{B}) = \mathbf{J}$ , we get that this current should induce a TMF,  $B_{\text{tor}} \sim RJ$ , where  $R \sim 10 \text{ km}$  is the stellar radius.

It should be noted that a compact star typically has a poloidal magnetic field  $B_{\text{pol}}$ , which is measured in astronomical observations. For instance, the radiation of a pulsar can be explained by the emission of electromagnetic waves by the rotating magnetic dipole associated with  $B_{\text{pol}}$ , provided there is a nonzero angle between  $\mathbf{B}_{\text{pol}}$  and  $\boldsymbol{\omega}$ . However, as shown in ref. [18], using general arguments for the magnetohydrodynamic equilibrium of an

axisymmetric NS, a purely poloidal magnetic field configuration turns out to be unstable. Thus an internal nonzero TMF should exist in a compact star.

Let us first consider the generation of TMF in NS composed of degenerate electrons and nucleons, like neutrons and protons. In this case  $u$  and  $d$  quarks are confined inside nucleons. The typical electron density in NS is  $n_e \approx 9 \times 10^{36} \text{ cm}^{-3}$ , which corresponds to the electron fraction  $Y_e \approx 0.05$ . It gives the chemical potential of electrons  $\mu_e \approx 125 \text{ MeV} \gg m_e$ . Therefore electrons are ultrarelativistic whereas neutrons and protons are nonrelativistic. Note that the nonzero electron mass was shown in ref. [19] to slightly contribute to the induced electric current. Thus we can assume that electrons are approximately massless in NS and the results of sections 3 and 4 are valid.

Since the chiral symmetry is unbroken, we can consider left and right chiral projections as independent degrees of freedom. For simplicity we shall take that left and right electrons are in equilibrium with  $\mu_L \sim \mu_R \sim \mu_e$ . The situations, when the chiral imbalance  $\mu_5 = (\mu_R - \mu_L)/2 \neq 0$  is important in NS, are studied, e.g., in refs. [4, 19]. Since  $n_p \ll n_n$  in NS,  $(V_L - V_R) = G_F n_n / \sqrt{2} \approx 12 \text{ eV}$  for  $n_n \approx 1.8 \times 10^{38} \text{ cm}^{-3}$ . Eventually, taking that  $\omega \sim 10^3 \text{ s}^{-1}$  and  $R \sim 10 \text{ km}$  as well as using eq. (4.8), we get that  $B_{\text{tor}} \approx 2.5 \times 10^8 \text{ G}$  can be generated in a rotating NS.

The obtained value of  $B_{\text{tor}}$  is comparable with  $B_{\text{pol}} \lesssim (10^8 - 10^9) \text{ G}$  in weakly magnetized old millisecond pulsars [20]. It should be noted that the stability of magnetic fields in NS can be reached if  $0 \leq E_{\text{tor}}/E_{\text{mag}} < 0.07$  [18], where  $E_{\text{tor}} \sim B_{\text{tor}}^2$  is the energy of TMF and  $E_{\text{mag}} \sim (B_{\text{tor}}^2 + B_{\text{pol}}^2)$  is the total magnetic energy. Our estimate for  $B_{\text{tor}}$  satisfies this criterion.

Let us discuss the creation of TMF in a hypothetical QS. Although QSs have not been observed yet, their properties are actively studied theoretically [21]. Various models of QS predict that it consists of free  $u$  and  $d$  quarks with some admixture of  $s$  quarks. After the analysis of various equations of state of QS matter, the strangeness fraction was found in ref. [22] not to exceed  $\sim 0.3$ . Thus we can approximately omit the contribution of  $s$  quarks in the calculation of the induced current.

We shall suppose that inside QS we have  $n_u = n_0/3 \approx 0.6 \times 10^{38} \text{ cm}^{-3}$  and  $n_d = 2n_0/3 \approx 1.2 \times 10^{38} \text{ cm}^{-3}$ , where  $n_0 \approx 1.8 \times 10^{38} \text{ cm}^{-3}$  is the nuclear density. At such high densities the chiral symmetry can be unbroken in QS [23], allowing one to consider the independent chiral projections of the wave functions of  $u$  and  $d$  quarks. Therefore we can again use the results of sections 3 and 4. As in case of NS, we can also assume that left and right quarks are in equilibrium,  $\mu_{u,d}^L \sim \mu_{u,d}^R \sim \mu_{u,d}$ , just for simplicity. Here  $\mu_u = (1/3)^{1/3} \mu_0 \approx 235 \text{ MeV}$  and  $\mu_d = (2/3)^{1/3} \mu_0 \approx 296 \text{ MeV}$ , where  $\mu_0 = 339 \text{ MeV}$ .

Finally, using, in eq. (4.8), the adopted values of densities and chemical potentials of quarks, the values of  $V_{L,R}$  in table 1 as well as for  $\omega \sim 10^3 \text{ s}^{-1}$  and  $R \sim 10 \text{ km}$ , we get that  $B_{\text{tor}} \approx 4.9 \times 10^8 \text{ G}$  can exist inside a rotating QS. The obtained value of TMF is slightly greater than  $B_{\text{tor}}$  for NS. Note that the derived strength of TMF is also in agreement with stability criterion obtained in ref. [18] for old weakly magnetized millisecond pulsars [20].

It should be noted that, in our estimate of TMF in QS, we account for only  $(ud)$  and  $(du)$  contributions to the electric current. Using the analogy of the rotating electroweak matter with the presence of an effective magnetic field [24] (see also eqs. (3.8) and (3.9) in section 3) and the results of ref. [25], we get that  $(uu)$  and  $(dd)$  interactions do not contribute to the current in eq. (4.8).

We should mention that, in the generation of TMF in a compact star, we discuss a thermally relaxed stage in the evolution of this astrophysical object. This approximation is

valid since we consider old millisecond pulsars with ages  $\sim (10^8 - 10^{10})$  yr [20]. It means that we discard any possible effects related to turbulence which should be treated on the basis of the Navier-Stokes equation. In our analysis we also do not consider a differential rotation either.

The creation of TMF in a compact star is closely related to the problem of the generation of the magnetic helicity defined as

$$H = \int d^3x (\mathbf{A} \cdot \mathbf{B}), \quad (5.1)$$

where  $\mathbf{A}$  is the 3D vector potential. If there is configuration of magnetic fields in a star consisting of toroidal and poloidal fields, then  $H$  in eq. (5.1) has the form [26],  $H = 2L\Phi_{\text{tor}}\Phi_{\text{pol}}$ , where  $\Phi_{\text{tor}}$  and  $\Phi_{\text{pol}}$  are the fluxes of toroidal and poloidal fields and  $|L| = 1$  is the linkage number. The magnetic helicity is a conserved quantity in a perfectly conducting medium. It is this fact which provides the stability of a poloidal field in a compact star. Note that another mechanism for the generation of the magnetic helicity in a nonrotating NS, based on the electron-nucleon electroweak interaction, was recently proposed in refs. [4, 27].

## 6 Pulsar kicks due to the asymmetric neutrino emission

In this section we shall use GRE for the description of the asymmetry in the neutrino emission from NS. We shall also consider the applicability of our results to explain linear velocities of pulsars.

It is well established that some pulsars have great linear velocities up to  $\sim 10^3 \text{ km}\cdot\text{s}^{-1}$  [8]. There are various models for the explanation of this phenomenon based on, e.g., the asymmetric electromagnetic radiation [28] and the asymmetric explosion leading to the anisotropic neutrino emission [29]. We also mention ref. [30], where the asymmetry in neutrino oscillations in matter and an external magnetic field was used to account for pulsar kicks. The idea that anisotropically emitted electrons, owing to CME, pass the momentum to NS, was discussed in ref. [19]. Nevertheless the origin of peculiar velocities of pulsars is still unclear.

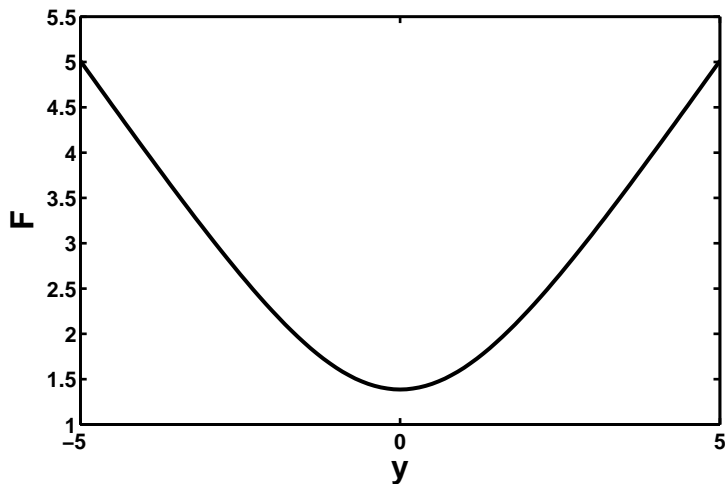
In ref. [9] it was established that linear velocities of pulsars are correlated with their angular velocities. Thus we may try to apply GRE, which predicts particle fluxes along the rotation axis, to explain pulsar kicks. However, unlike ref. [19], we shall examine the possibility of asymmetric neutrino emission since it is not very clear how charged particles can escape NS.

Using eqs. (4.4) and (4.5), we get the total hydrodynamic current of neutrinos along the rotation axis as

$$j_{\text{L}}^3 = G_{\text{F}} n_n \frac{\omega T}{\pi \sqrt{2}} F(y), \quad F(y) = \int_0^\infty \left( \frac{1}{e^{x-y} + 1} + \frac{1}{e^{x+y} + 1} \right) dx = 2 \ln(1 + e^y) - y, \quad (6.1)$$

where  $y = (\mu_{\text{L}} - V_{\text{L}})/T$  and  $T$  is the neutrino temperature. To derive eq. (6.1) we take into account that  $V_{\text{L}} \approx -G_{\text{F}} n_n / \sqrt{2}$  for  $\nu_e$  (see table 1) and  $n_n \gg n_e$ . Note that both  $\mu_{\text{R}}$  and  $V_{\text{R}}$  are equal to zero for ultrarelativistic neutrinos. That is why we account for the contribution of only left neutrinos in eq. (6.1).

The more complicated structure of the hydrodynamic current in eq. (6.1) compared to that of the electric current in eq. (4.8) is owing to the fact that  $j_{\text{L}\bar{f}}^3$  has the same direction as  $j_{\text{L}f}^3$ , whereas  $J_{\text{L}\bar{f}}^3$  is directed oppositely to  $J_{\text{L}f}^3$ . The function  $F(y)$  is plotted in figure 1.



**Figure 1.** The function  $F$  in eq. (6.1) versus  $y$ .

It is interesting to mention that  $F(0) \approx 1.4$ . Thus, there is a nonzero neutrino flux even at  $\mu_L = 0$ .

The simulations carried out in ref. [31] show that there is a significant nonzero neutrino asymmetry  $\Delta n_{\nu_e} = n_{\nu_e} - n_{\bar{\nu}_e}$ , owing to direct Urca processes, which lasts up to  $t_1 \sim 0.1$  s after the onset of the supernova collapse. When  $0 < t < t_1$ , the typical neutrino energy  $E_\nu \sim 10$  MeV and chemical potential of  $\nu_e$  is  $\mu_L \sim 10$  MeV [32]. At  $t_1 < t \lesssim t_2 \sim 10^2$  yr other neutrino species start to be emitted resulting in the diminishing of  $\mu_L$ . When  $t_2 \lesssim t \lesssim t_3 \sim 10^6$  yr only  $\nu\bar{\nu}$  pairs can be emitted in modified Urca processes [33] leading to  $\mu_L = 0$ . For simplicity we shall assume that  $T \sim 10^8$  K = const and  $E_\nu \sim 10$  keV [33] at this stage of the NS evolution.

The total momentum carried away by neutrinos during the time interval  $\Delta t$  is  $P \sim j^3 E_\nu S \Delta t$ , where  $S = \pi R^2$  is area of the equatorial cross section of NS and  $R \sim 10$  km is the NS radius. Using eq. (6.1), one gets that the greatest  $P$  is achieved at  $t_2 \lesssim t \lesssim t_3$ . As a result, NS will get a recoil velocity  $v = P/M$ , where  $M$  is the NS mass. We shall take  $M = 1.44 M_\odot$ , where  $M_\odot \approx 2 \times 10^{33}$  g is the solar mass, to get the upper bound for  $v$ . Taking that  $\omega \sim 10^3$  s $^{-1}$ , we get that  $v \sim 10^{-16}$  cm  $\cdot$  s $^{-1}$ . The obtained value is sure to be beyond the possibility of modern astronomical observations. Therefore, despite there is a nonzero anisotropy in the neutrino emission in NS owing to GRE, this effect is unlikely to result in any testable phenomena.

## 7 Discussion

In conclusion we note that in the present work we have studied the evolution of particles, involved in the parity violating electroweak interaction, in the rotating matter. In section 3, we have obtained the new exact solution of the Dirac equation for a test ultrarelativistic particle, which account for the noninertial effects. Then, in section 4, we have computed the induced electric current along the rotation axis on the basis of the exact solution of the Dirac equation. In section 5, we have applied our results for the generation of TMF and the magnetic helicity in compact rotating stars. Finally, in section 6, we have considered the

production of the anisotropy in the neutrino emission from NS, owing to GRE, and examined the applicability of this effect for the explanation of peculiar velocities of pulsars.

Several new results have been obtained in this work. Firstly, we mention that the vierbein vectors in eq. (3.3) have never been used previously in the Dirac eq. (3.2), which accounts for the electroweak interaction with background matter in curved space-time. Another vierbein was recently used in ref. [11]. However, the choice of the vierbein in the present work is likely to be more appropriate for ultrarelativistic particles in a rotating frame. In particular, here we have obtained the correct form of the “centrifugal” energy, or the energy of the rotation—angular momentum coupling,  $E_{\text{cf}} = -(\mathbf{J} \cdot \boldsymbol{\omega})$ ; cf. eq. (3.13). The obtained expression for  $E_{\text{cf}}$  coincides with the result of ref. [34] derived on the basis of the general analysis. The form of  $E_{\text{cf}}$  obtained in ref. [11], where another vierbein was used, is slightly different. Therefore, the vierbein adopted in ref. [11] is likely to be more appropriate for the description of nonrelativistic particles in a rotating frame; cf. ref. [35].

Secondly, we have predicted the new GRE. This effect consists in the appearance of the electric current in the rotating matter composed of massless particles involved in the parity violating electroweak interaction. This electric current flows along the rotation axis. The new GRE is analogous to CME, known in QED, which consists in the generation of the electric current of massless charged particles along the external magnetic field [2, 3]. It should be noted that, for the first time, the analogy between the motion in a rotating electroweak matter and in an external magnetic field was mentioned in ref. [24].

Note that the appearance of the electric tension in a rotating conductor owing to the noninertial effects was also discussed in ref. [36]. The electric tension, predicted in ref. [36], is induced mainly by the Coriolis force acting on charged particles in a rotating conductor. If  $\boldsymbol{\omega}$  is chosen along the  $z$ -axis, this tension is found in ref. [36] to be along the azimuthal direction. In our case, the electric current is owing to both the matter rotation and the presence of the parity violating electroweak interaction. We predict that the induced electric current flows along the rotation axis.

We have used the solution of a Dirac equation in the rotating frame for the calculation of the induced electric current. It means that this current flows inside the rotating matter since the quantum states of charged particles are measured by a corotating observer. For example, if one used the wave functions obtained in ref. [37], although they look similar to those in eqs. (3.19) and (3.20), we would get the electric current with respect to a nonrotating observer, which cannot be applied for the generation of the internal TMF.

We have used the calculated electric current to generate TMF and the magnetic helicity inside a rotating compact star, like NS or QS. The strength of TMF generated turned out to be moderate,  $B_{\text{tor}} \gtrsim 10^8$  G, for both NS and QS. However, such TMF is comparable with a poloidal field in weakly magnetized old millisecond pulsars [20]. Note that the obtained strength of TMF is in agreement with a criterion for the magnetic field stability derived in ref. [18]. It should be noted that our model for the generation of TMF does not require the existence of a significant chiral imbalance between left and right charged particles. Such an imbalance is essential if CME is used to create TMF; cf. ref. [19].

Finally, in section 6, we analyzed the possibility to apply GRE to explain linear velocities of pulsars by the asymmetric neutrino emission from a rotating NS. We have estimated the total neutrino flux along the rotation axis as well as the recoil velocity of NS. It turned out that a pulsar kick caused by GRE is outside the observationally tested region.

At the end of this section we should make a comment on the influence of nonzero masses of charged particles on the generation of an electric current in a rotating star. It was



mentioned in ref. [19] that the value of the current, induced by CME, slightly diminishes if a small, compared to the energy, but nonzero electron mass is accounted for. A more detailed analysis of the influence on the current, induced in frames of CME, from nonzero masses of flavored fermions, in case of strong interactions, was made in ref. [38]. It was shown in ref. [38] that CME disappears only in the great masses limit. The effect of the nonzero electron mass on the generation of strong magnetic fields in magnetars was also studied in ref. [39].

We should mention that, in case of GRE, a nonzero current in eq. (4.8) exists even at zero chiral imbalance:  $\mu_5 = 0$ . Therefore, even if an initial  $\mu_5(0) \neq 0$  is washed out owing to spin-flip processes taking place at a nonzero mass, as predicted in refs. [4, 27, 39], the current will be nonzero due to  $V_5 = (V_L - V_R)/2 \neq 0$ . For example, in section 5, we assumed that  $\mu_5 = 0$  to simplify the estimates. In the model for the generation of strong magnetic fields in magnetars, elaborated in refs. [4, 27],  $\mu_5 \sim -V_5$  appears inside NS in the course of the magnetic fields evolution. In this case, our estimates for  $B_{\text{tor}}$  obtained in section 5 will slightly change since  $\mu_R \neq \mu_L$ .

GRE at  $V_5 \neq 0$  and  $\mu_5 = 0$  is likely to exist in all particular cases we considered in the present work since we discussed ultrarelativistic particles having  $m_f/\langle E_f \rangle \ll 1$ , where  $m_f$  is the fermion mass and  $\langle E_f \rangle$  is the typical fermion energy. Nevertheless, a more detailed quantum field theory analysis of this fact, like in ref. [38], is required.

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## A Details of the electric current calculation

In this appendix we derive the electric current in eq. (4.8) and discuss the approximations made.

Let us first consider left fermions. Using eq. (4.1), one gets that the contribution of particles and antiparticles to the electric current along the rotation axis is

$$J_L^3 = q_f \sum_{N,s=0}^{\infty} \int dp_z \left[ \bar{\psi}_f \gamma^\mu(x) \psi_f \rho_f^L(E_f) - \bar{\psi}_{\bar{f}} \gamma^\mu(x) \psi_{\bar{f}} \rho_{\bar{f}}^L(E_{\bar{f}}) \right], \quad (\text{A.1})$$

where  $E_{f,\bar{f}} = \mathcal{E} \pm V_L \pm \omega \text{sgn}(V_L)(l - 1/2)$  are the energies of particles and antiparticles, which also include  $E_{\text{cf}} = -(\mathbf{J} \cdot \boldsymbol{\omega})$ , and  $\psi_{f,\bar{f}}$  are the wave functions of particles and antiparticles.

Assuming that  $\mathcal{E} \gg |E_{\text{cf}}|$ , we can expand the distribution function in eq. (A.1) in a series

$$\rho_f^L(E_f) = \rho_f^L(E_0) - \frac{\omega \text{sgn}(V_L) l'}{T} \frac{\exp(E_0)}{(\exp(E_0) + 1)^2} + \dots, \quad (\text{A.2})$$

where  $E_0 = \mathcal{E} \pm V_L \mp \omega \text{sgn}(V_L)/2$  and  $l' = l$  for the upper sign in  $E_0$  or  $l' = l - 1$  for the lower sign in  $E_0$ . One can write down the decomposition of  $\rho_{\bar{f}}^L(E_{\bar{f}})$  analogous to that in eq. (A.2).

Using the explicit form of the wave functions in eqs. (3.19) and (3.20) and the known sum involving the Laguerre function [40],

$$\sum_{s=0}^{\infty} (s - N) I_{N,s}^2(\rho) = \rho, \quad (\text{A.3})$$

we get that accounting for  $E_{\text{cf}}$  is equivalent to the replacement  $\mu_L \rightarrow \mu'_L = \mu_L + (\omega r)^2 V_L$ . Here we neglected terms  $\sim \omega^3$ . In fact, the terms  $\sim (\omega r)^2$  are the noninertial contributions to the electric current.

Thus the ground state  $N = 0$  contribution to  $J_L^3$  in eq. (A.1) takes the form,

$$J_L^3 = -q_f \frac{\omega V_L}{\pi} \int dp \left[ \rho_f^L(p + V_L) - \rho_f^L(p - V_L) \right], \quad (\text{A.4})$$

where we should use  $\mu'_L$  instead of  $\mu_L$ . Introducing the new variables  $x = p/T$  and  $y = (\mu_L - V_L)/T$  in eq. (A.4) and using the identity

$$\int_0^\infty \left( \frac{1}{e^{x-y} + 1} - \frac{1}{e^{x+y} + 1} \right) dx = y, \quad (\text{A.5})$$

and eq. (A.4), we get that  $J_L^3 = -(q_f/\pi)\omega V_L(\mu'_L - V_L)$ . The contribution of right fermions can be obtained analogously. It is interesting to mention that the electric current obtained does not depend on the plasma temperature.

In section 5 we use the following values of the parameters:  $\mu_{L,R} \sim 10^2 \text{ MeV}$ ,  $V_{L,R} \sim 10 \text{ eV}$ ,  $\omega \sim 10^3 \text{ s}^{-1}$ , and  $r < R \sim 10 \text{ km}$ . Therefore, one gets that  $\mu_{L,R} \gg V_{L,R} \gg (\omega r)^2 V_{L,R}$ . Taking into account these estimates we arrive to eq. (4.8).

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